Exercise 34

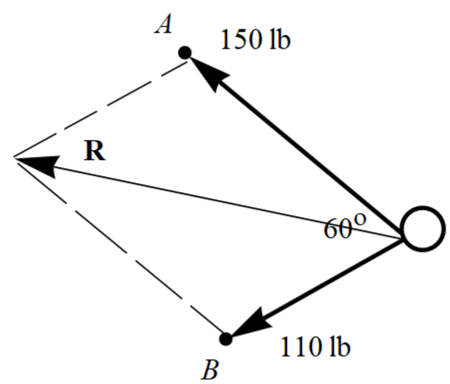
Two persons pull horizontally on ropes attached to a post, the angle between the ropes being 60°. Person A pulls with a force of 150 lb, while person B pulls with a force of 110 lb.

(a) The resultant force is the vector sum of the two forces. Draw a figure to scale that graphically represents the three forces.

(b) Using trigonometry, determine formulas for the vector components of the two forces in a conveniently chosen coordinate system. Perform the algebraic addition, and find the angle the resultant force makes with A.

Solution

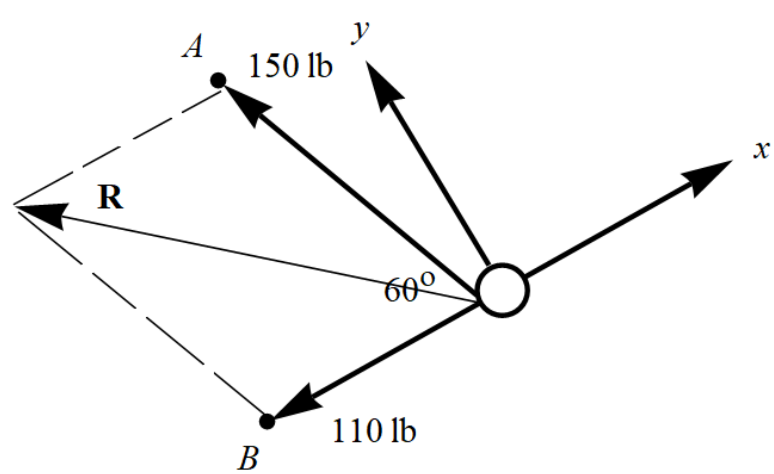
Part (a)



The vector **R** is the resultant force and is the vector sum of the 150 lb and 110 lb forces.

Part (b)

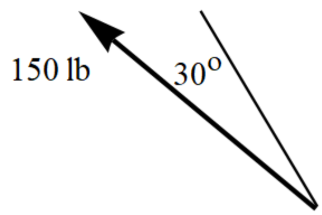
Use the xy-coordinate system shown below.



Let **A** and **B** be the forces due to person A and person B, respectively. Since **B** points exclusively in the negative x-direction, it has no y-component, and a minus sign is needed in the x-component.

$$\mathbf{B} = (-110, 0)$$

Zoom in on **A** in the figure.

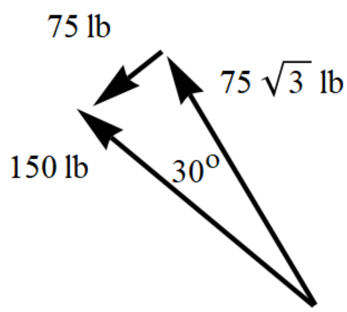


The angle between **A** and the y-axis is 30°. Determine the components of this vector.

$$\cos 30^\circ = \frac{y}{150}$$
$$\sin 30^\circ = \frac{x}{150}$$

Solve for x and y.

$$y = 150\cos 30^\circ = 75\sqrt{3} \text{ lb}$$
$$x = 150\sin 30^\circ = 75 \text{ lb}$$



Since the 75 lb force points in the negative x-direction, a minus sign is needed. On the other hand, the $75\sqrt{3}$ lb force points in the positive y-direction, so no minus sign is needed.

$$\mathbf{A} = (-75, 75\sqrt{3}) \text{ lb}$$

Consequently, the resultant in this coordinate system is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (-110, 0) + (-75, 75\sqrt{3}) = (-185, 75\sqrt{3}).$$

Determine the angle of **R** from the x-axis.

$$\tan\theta = \frac{75\sqrt{3}}{-185} \quad \to \quad \theta = \pi - \tan^{-1}\left(\frac{75\sqrt{3}}{185}\right) \approx 144.9^\circ$$

Therefore, the angle between **R** and **A** is about $144.9^\circ - 30^\circ - 90^\circ = 24.9^\circ$.